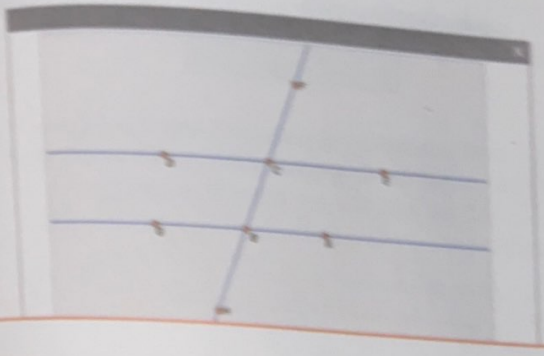
Angle Relationships and Parallel Lines

I Can... use the relationships between angles to find the measures of missing angles.

What Vocabulary Will You Learn?
alternate exterior angles
alternate interior angles
corresponding angles
exterior angles
interior angles
parallel lines
perpendicular lines
transversal

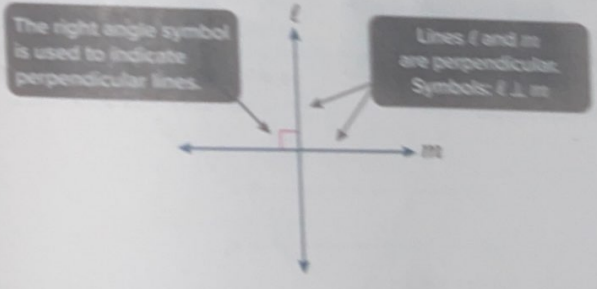
Explore Parallel Lines and Transversals

Online Activity You will use Web Sketchpad to explore the relationships between angles created by parallel lines and transversals.



Learn Lines, Angles, and Transversals

Pairs of angles can be classified by their relationship to each other. A special case occurs when two lines intersect in a plane to form a right angle. These lines are **perpendicular lines**. Special notation is used to indicate perpendicular lines. Read $\ell \perp m$ as *line ℓ is perpendicular to line m.*

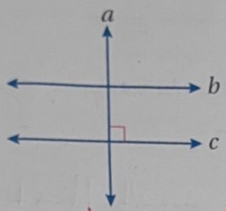


(continued on next page)

Your Notes

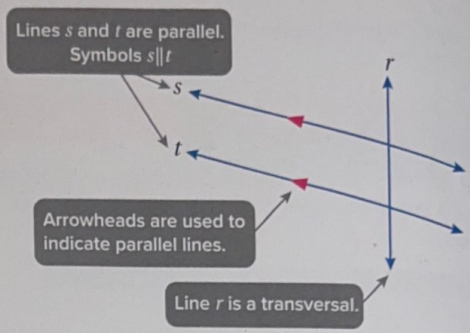
Talk About It!

If the transversal is perpendicular to one of the parallel lines, what relationship does the transversal have to the other parallel line?



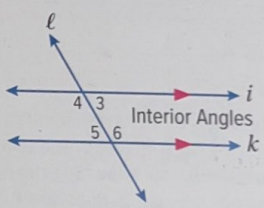
If a transversal is perpendicular to one of the parallel line then it is also perpendicular to the other parallel line

Two lines in a plane that never intersect are called **parallel lines**. A line that intersects two or more other lines in a plane is called a **transversal**. Special notation is used to indicate parallel lines. Read $s \parallel t$ as *line s is parallel to line t.*



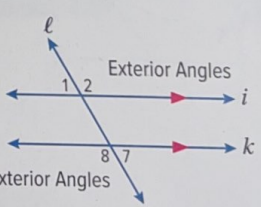
When a transversal intersects two parallel lines, eight angles are formed. Four of the angles are **interior angles**, located in the space between the parallel lines, and four are **exterior angles** that lie outside the parallel lines.

Interior angles lie inside the parallel lines.



Examples: ∠3, ∠4, ∠ 5 , ∠ 6

Exterior angles lie outside the parallel lines.

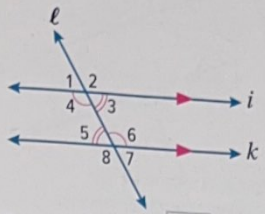


Examples: ∠1, ∠2, ∠ 7 , ∠ 8

(continued on next page)

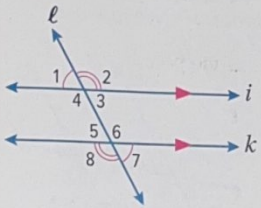
When two parallel lines are cut by a transversal, there is a relationship between the angles that are created. The angles in certain angle pairs, **alternate interior angles**, **alternate exterior angles**, and **corresponding angles**, have the same angle measure.

Alternate interior angles are interior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal.



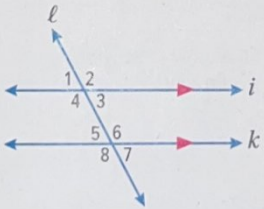
Examples: $m\angle 4 = m\angle 6$ and $m\angle 3 = m\angle$ 5

Alternate exterior angles are exterior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal.



Examples: $m\angle 1 = m\angle 7$ and $m\angle 2 = m\angle$ 8

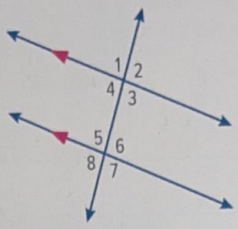
Corresponding angles are those angles that are in the same position on the two lines in relation to the transversal. When the lines are parallel, their measures are equal.



Examples: $m\angle 1 = m\angle 5$, $m\angle 2 = m\angle 6$, $m\angle 3 = m\angle 7$, and $m\angle 4 = m\angle$ 8

Example 1 Classify Angle Pairs

Classify the relationship between ∠1 and ∠7 in the figure as *alternate interior, alternate exterior,* or *corresponding.*



∠1 and ∠7 are exterior angles that lie on opposite sides of the transversal. They are alternate exterior angles.

Think About It!
What are the locations of the angles with respect to the parallel lines?

exterior or outside of the parallel lines

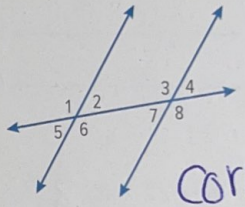
Talk About It!
Name another pair of alternate exterior angles. How many pairs of alternate exterior angles are there when two parallel lines are cut by a transversal? Will this happen when any two parallel lines are cut by a transversal? Explain.

∠2 and ∠8

There will always be two pairs of alternate exterior angles when two parallel lines are cut by a transversal because there will be four exterior angles total

Check

In the figure, the two lines shown are parallel and intersected by a transversal. Classify the relationship between ∠2 and ∠4 as *alternate exterior, alternate interior,* or *corresponding.*



corresponding

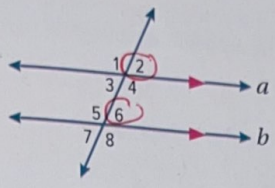
Go Online You can complete an Extra Example online.

Pause and Reflect

How can the meaning of the words *alternating* and *corresponding* help you think about alternate interior, alternate exterior, and corresponding angles?

Example 2 Classify Angle Pairs

Classify the relationship between ∠2 and ∠6 in the figure as *alternate interior*, *alternate exterior*, or *corresponding*.

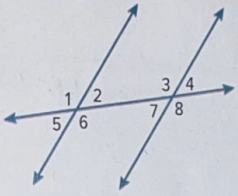


∠2 and ∠6 are in the same position on the two lines in relation to the transversal. They are corresponding angles.

Check

In the figure, the two lines shown are parallel and intersected by a transversal. Classify the relationship between ∠4 and ∠5 as *alternate exterior*, *alternate interior*, or *corresponding*.

alternate exterior angles



Go Online You can complete an Extra Example online.

Think About It!

What are the locations of the angles with respect to the parallel lines?

above, or on the same side of the parallel lines

Talk About It!

Name another pair of corresponding angles. How many pairs of corresponding angles are there when two parallel lines are cut by a transversal? Will this happen when any two parallel lines are cut by a transversal? Explain.

∠1 and ∠5

There will always be four pairs of corresponding angles, because there are eight angles formed and each one will be paired with corresponding angles

Pause and Reflect

How did what you know about *alternate interior*, *alternate exterior*, and *corresponding angles* help you solve the problem?

Record your observations here

أجيال المستقبل

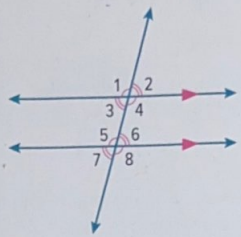
Mrs| Aya Hameed

Learn Find Missing Angle Measures

When two parallel lines are cut by a transversal, eight angles are formed. Special relationships exist among pairs of angles.

Go Online Watch the video to learn how to use these relationships to find the measure of any angle formed by two parallel lines and a transversal.

The video shows the following parallel lines.

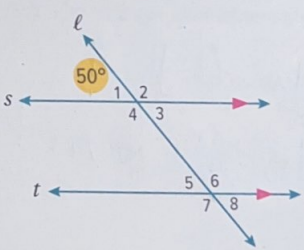


Complete the missing angle measures in the table.

Angle	1	2	3	4	5	6	7	8
Measure	105°	75°	75°	105°	105	75	75	105

If you know the measure of one angle, you can use your knowledge of supplementary and vertical angles to find the measures of the three angles that are along the same line.

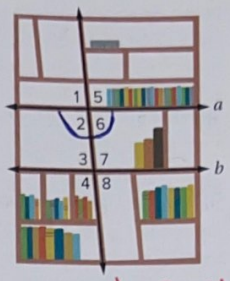
In the figure below, suppose $m\angle 1 = 50°$.
$m\angle 2 = 130°$ because $\angle 1$ and $\angle 2$ are supplementary
$m\angle 3 = 50°$ because $\angle 1$ and $\angle 3$ are vertical angles.
$m\angle 4 = 130°$ because $\angle 1$ and $\angle 4$ are supplementary



Talk About It!
Once you know the measures of angles 2, 3, and 4, how can you find the measures of angles 5, 6, 7, and 8?

Example 3 Find Missing Angle Measures

Mrs. Kumar designed the bookcase shown. Line a is parallel to line b.



If $m\angle 2 = 105°$, find $m\angle 6$ and $m\angle 3$. Justify your answer.

Part A Find $m\angle 6$.

Since $\angle 2$ and $\angle 6$ are supplementary, the sum of their measures is 180°.

So, $m\angle 6 = 180° - 105°$ or 75°.

m<2 and <5 vertical angles
m<2 and <4 corresponding Angle
m<2 and <7 alternate interior

Part B Find $m\angle 3$.

Angle 6 and $\angle 3$ are interior angles that lie on opposite sides of the transversal. Since they are alternate interior angles, their measures are equal

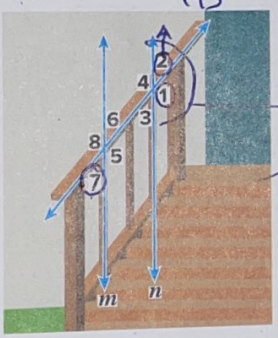
So, $m\angle 3 = 75°$.

Check

Arianna's house has the porch stairs shown. Line m is parallel to line n. If $m\angle 7 = 35°$, find $m\angle 1$ and $m\angle 2$.

145

m<1=145
m<7=35



→ 180-35 = 145

Show your work here

حل المستقيل

Done by/ Mrs Aya Hamed

Think About It!

Think about the special relationship between $\angle 2$ and $\angle 6$. How does $\angle 3$ relate to either $\angle 2$ or $\angle 6$?

Talk About It!

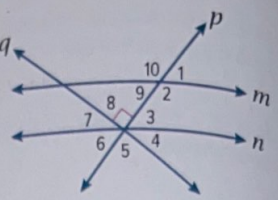
How many unique angle measurements exist in the figure?

Because the two lines are parallel and intersect by a transversal the right angles that are formed have two unique angle measure 75° and 105°

Go Online You can complete an Extra Example online.

Example 4 Find Missing Angle Measures

In the figure, line m is parallel to line n, and line q is perpendicular to line p. The measure of $\angle 1$ is 40°.



What is the measure of $\angle 7$?

Step 1 Find $m\angle 6$.

Study the figure. Angle 7 is adjacent to angle 6 and angles 1 and 6 form a special angle pair. Find $m\angle 6$ first. Then use $m\angle 6$ to find $m\angle 7$.

Because $\angle 1$ and $\angle 6$ are alternate exterior angles, their measures are equal. The $m\angle 1$ is 40°, so the $m\angle 6 = 40°$.

Step 2 Find $m\angle 7$.

Becausee $\angle 6$, $\angle 7$, and $\angle 8$ form a straight line, the sum of their measures is 180°.

$m\angle 6 + m\angle 7 + m\angle 8 = 180°$	Write the equation.
$40 + m\angle 7 + 90 = 180°$	Replace $m\angle 6$ with 40° and $m\angle 8$ with 90°.
$130 + m\angle 7 = 180°$	Add.
$-130° \quad = -130°$	Subtraction Property of Equality
$m\angle 7 = 50$	Simplify.

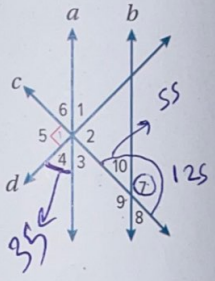
So, $m\angle 7$ is 50°.

Check

In the figure, line a is parallel to line b, and line c is perpendicular to line d. The measure of $\angle 7$ is 125°. What is the measure of $\angle 4$?

Show your work here

$\angle 4 = 35°$



Think About It!

If you know $m\angle 8$, and want to find the measure of $\angle 7$, what other angle will help you?

Talk About It!

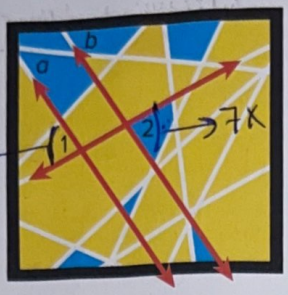
If $m\angle 1 = 40°$, do you have enough information to find all of the missing angles in the figure? Explain.

yes, The special relashionships between angles created by parralel line are transversal along with vertical and supplementary relashipps allow all of the missing angles

Go Online You can complete an Extra Example online.

Check

In the painting, line a is parallel to line b. The measure of angle 1 is $(5x + 24)°$ and the measure of angle 2 is $7x°$. Find $m\angle 1$.



5x+24

Show your work here

$\angle 1 = \angle 2$ alternate exterior

$5x + 24 = 7x$

$-5x \quad -5x$

$\frac{24}{2} = \frac{2x}{2}$

$x = 12$

$\angle 1 = 5x + 24$

$5(12) + 24 = 84°$

Go Online You can complete an Extra Example online.

Pause and Reflect

What have you learned about the angles formed by *parallel lines* and *transversals*? Can you name the angles that are formed? Can you determine which angles have the same measure?

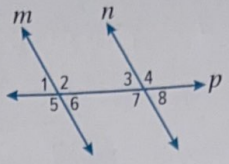
Record your observations here

Name Done by Mrs/ Aya Hamed Period

Practice

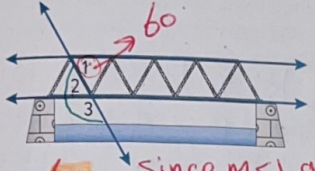
Go Online You can complete your homework online.

For Exercises 1–4, use the figure at the right. In the figure, line *m* is parallel to line *n*. For each pair of angles, classify the relationship in the figure as *alternate interior*, *alternate exterior*, or *corresponding*. (Examples 1 and 2)



1. ∠2 and ∠7 alternate interior
2. ∠1 and ∠3 corresponding
3. ∠4 and ∠5 alternate exterior
4. ∠5 and ∠7 corresponding

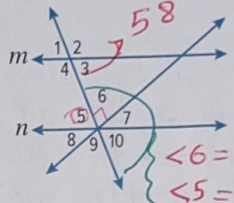
5. Arturo is designing a bridge for science class using parallel supports for the top and bottom beam. Find $m\angle 2$ and $m\angle 3$ if $m\angle 1 = 60°$. Justify your answer. (Example 3)



m<2 = 60 since m<1 and <2 are alternate interior they are equal

m<2 and m<3 are supplementary so m<2 + <3 = 180 m<3 = 120

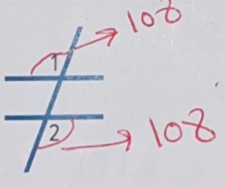
6. In the figure, line *m* is parallel to line *n*. The measure of ∠3 is 58°. What is the measure of ∠7? (Example 4)



<6 = 90
<5 = 58
<7 + <6 + <5 = 180 <7 = 32

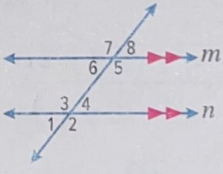
Test Practice

7. The symbol below is an equal sign with a slash through it. It is used to represent *not equal to* in math, as in $x \neq 5$. If $m\angle 1 = 108°$, classify the relationship between ∠1 and ∠2. Then find $m\angle 2$. Assume the equal sign consists of parallel lines.



alternate exterior angles
m<2 = 108

8. **Multiselect** In the figure, line *m* and line *n* are parallel. Select all of the statements that are true.



- [x] ∠1 and ∠8 are alternate exterior angles.
- [x] ∠3 and ∠7 are corresponding angles.
- [ ] ∠2 and ∠8 are corresponding angles.
- [x] ∠4 and ∠6 are alternate interior angles.
- [ ] ∠5 and ∠7 are corresponding angles.

Apply

9. Angles A and B are corresponding angles formed by two parallel lines cut by a transversal. If $m\angle A = 4x°$ and $m\angle B = (3x + 7)°$, find the value of x. Explain.

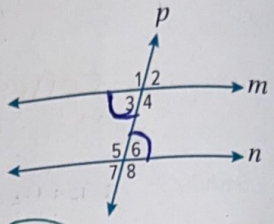
$x = 7$, Corresponding angles are congrent

$4x = 3x + 7$

$-3x \quad -3x$

$x = 7$

10. In the figure, line m is parallel to line n. If $m\angle 3 = (7x - 10)°$ and $m\angle 6 = (5x + 10)°$, what are the measures of $\angle 3$ and $\angle 6$?



$m\angle 3 = m\angle 6$ alternate interior

$7x - 10 = 5x + 10$

$+10 \quad +10$

$m\angle 6 = (5 \times 10) + 10 = 60$

$m\angle 3 = (7 \times 10) + 10 = 60$

$7x = 5x + 20$

$-5x \quad -5x$

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

11. MP **Reason Abstractly** Refer to the figure in Exercise 10. Look at a pair of angles described as *interior angles on the same side of the transversal*. What do you think the relationship is between these angles? Explain why you think this is true.

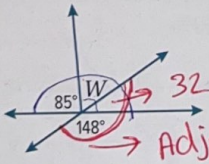
Interior angles that are on the same side of the transversal are supplementary. one of the interior angles is supplementary to the corresponding angle of the other interior

12. Determine if the statement is *true* or *false*. Construct an argument that can be used to defend your solution.

If a transversal intersects two parallel lines, the measures of the alternate exterior angles are equal.

True, Alternate exterior angles are equal because one of the those angles correspondy angles are equal and vertical

13. Determine the measure of $\angle W$. Construct an argument that can be used to defend your solution.



Adjecent

$85 + W + 32 = 180$

$117 + W = 180$

$-117 \quad -117$

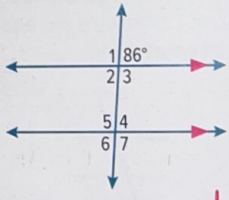
$W = 63$

$148 + x = 180$

$-148 \quad -148$

$x = 32$

14. MP **Find the Error** A student was finding the measure of $\angle 5$ in the figure below. She concluded that $m\angle 5 = 86°$ because it is a corresponding angle with $\angle 2$. Find her mistake and correct it.



$\angle 5$ is a corresponding angles with $\angle 1$ $m\angle 1$ is supplementary to 86° so $m\angle 1 = 94$ Because corresponding angles are equal